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QUARK-MASS EFFECTS IN SCHEME-INVARIANT PERTURBATION THEORY

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We report the results of massive generalization of the scheme-invariant perturbation theory that represents physical observables in terms of renormalization-scheme invariant quantities. For the functions depending on one invariant argument expressions which explicitly incorporate threshold effects are given. Analogous generalizations of equations relating scale parameters for different processes are obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Массовые эффекты в схемно-инвариантной теории возмущений

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Представлены результаты массивного обобщения схемно-инвариантной теории возмущений, которая выражает физические наблюдаемые в терминах инвариантов схемы перенормировки. Для функций, зависящих от одного инвариантного аргумента, получены выражения, явным образом включающие пороговые эффекты. Приведены аналогичные обобщения уравнений, связывающих шкалы различных процессов.

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1. Introduction

The results of renormalized perturbation theory as they are as well as improved by the renormalization group method (RGM)[1,2] contain explicit dependence on the renormalization scheme employed. The problem of scheme dependence (SD) becomes important in QCD due to a rather large value of the effective coupling constant in the physical region of interest.

Recently this SD problem was attacked from a new standpoint[3,4] which is equivalent to the introduction of a number of coupling constants each of which is attached to a single physical process (or situation). Each quantum-field function or matrix element can then be expressed in a scheme-independent (SI) way in terms of its "own" coupling constant or scale parameter. The unity of the theory is provided by the set of SI relation between scale parameters of different processes.

All this formalism was developed [4,5] in the massless case for single-argument objects within the RG technique. We shall call it the scheme-invariant perturbation theory (SIPT). The present paper contains the results of massive generalization of this SIPT.

The account of mass dependences can be made on the basis of RGM technique in the massive case developed by one of the authors several years ago [6]. The foundations of such massive RGM were formulated in 50-ies [1,2]. For example, the one-loop mass-dependent contributions to the invariant (effective) coupling constant, in a full analogy with massless case, are summed into a geometric progression. Technically this is achieved by solving the differential RG equations (first obtained in [1]) containing a fixed renormalized pole mass. The above-mentioned geometric progression is an exact solution [7] of this differential equation.

A two-loop solution of the mass-dependent equation for the coupling has been obtained in [6] and has the form

$$\bar{a}^{(2)}_{RG} = \frac{a}{1 - aA_1 + a\frac{A_2}{A_1}\ln(1 - aA_1)}, \qquad (1)$$

where $A_\ell = A_\ell(Q^2, m^2, \mu^2)$ is a genuine 1-loop contribution to the PT expression: $\bar{a}_{PT} = a + a^2A_1 + a^3(A_1^2 + A_2) + O(a^4)$.

Note that this result, on the one hand, contains only (mass-dependent) coefficients A_ℓ of perturbation series and at the same time in the pure logarithmic regions (when $A_\ell \simeq \beta_\ell \ln Q^2/\mu^2$) coincides with the well-known ultraviolet 2-loop expression for $\bar{a}$ (see, e.g., Eq. (34) in [2] or Eq. (43.12) in [8]).

Analogous expressions were obtained [6] for other single-argument functions possessing anomalous dimensions, e.g., for the moments of structure functions, of effective masses. In the one-loop approximation ($\Gamma_{PT} = 1 + aS_1(Q^2, m^2, \mu^2) + O(a^2)$)

$$\Gamma_{RG} = \left(\frac{\bar{a}_{RG}}{a}\right)^{S_1/A_1} = [1 - aA_1(Q^2, m^2, \mu^2)]^{-n(Q^2)},$$

$$n(Q^2) = \frac{S_1(Q^2, m^2, \mu^2)}{A_1(Q^2, m^2, \mu^2)}. \qquad (2)$$

Eqs.(1) and (2) are a generalization of the corresponding massless eqs. Only for $m \to 0$, when $S \to \gamma_1 \ln Q^2/\mu^2$, the exponent n becomes Q^2-independent.

Eqs.(1) and (2) were obtained in the MOM-scheme. They can be reformulated in any other scheme, but will still be SD. Our aim is to avoid this SD. With this purpose we use the approach of ref./3, 4, 5/. Below we give a short review of this approach.

2. Scheme-Invariant Perturbation Theory

Consider a single-argument function R possessing zero anomalous dimension with the perturbative expansion

$$R_{PT}(Q^2, \ldots, a) = a_i \{1 + a_i r_1^i (Q^2, \ldots) + a_i^2 r_2^i (Q^2, \ldots) \ldots \}, \tag{3}$$

where the expansion parameter $a_i = \alpha_i/\pi$ as well as coefficients r_ℓ^i are SD, i being the index of the scheme. Differentiating eq.(3) with respect to $\ln Q^2$ and on the other hand, solving eq.(3) for a_i, i.e., expressing a_i in terms of a series over R, we come to the differential equation for R:

$$R' = F(R) = \sum_{\ell \geq 1}^{\ell_{max}} R^{\ell+1} f_\ell, \quad (R' \equiv Q^2 \frac{dR}{dQ^2}) \tag{4}$$

which is explicitly SI. That means that all f_ℓ = SI. Note that in the massless case all f_ℓ are constant.

Solution of eq.(4) can be given in the form

$$\beta_1 \ln \frac{Q}{\Lambda_R^2} = \frac{1}{R} - \frac{\beta_2}{\beta_1} \ln(1 + \frac{\beta_1}{\beta_2 R}) + I(R), \tag{5}$$

where $\beta_1 = \frac{11 - \frac{2}{3} f}{4}$ and $\beta_2 = \frac{102 - \frac{38}{3} f}{16}$ are one- and two-loop β-function coefficients respectively, and $I(R) = 0$ in the two-loop approximation. Here Λ_R is an integration constant which is directly connected with a given quantity i.e., with a given physical process. The relation between Λ_{SC} of any renormalization scheme and Λ_R is:

$$\beta_1 \ln (\frac{\Lambda_R}{\Lambda_{SC}})^2 = r_1^{SC}. \tag{6}$$

At the same time the scales of different processes, say A and B are connected by the SI relation

$$\beta_1 \ln\left(\frac{\Lambda_A}{\Lambda_B}\right)^2 = r_{1A} - r_{1B} \tag{7}$$

valid in any order of PT.

To illustrate the application of SIPT, consider two examples of a perturbative calculation in QCD:

1) Υ-decay. The physical observable here is the ratio of widths[9]:

$$R_\Upsilon = \frac{4\alpha}{5\pi}\,\frac{\Gamma(\Upsilon \to ggg)}{\Gamma(\Upsilon \to \gamma gg)} = a(M_\Upsilon)[1 + (7.85 - 0.61f)a(M_\Upsilon)],$$

2) J/ψ hyperfine splitting[9]

$$R_\psi = \frac{\alpha^2}{2\pi}\,\frac{\Delta E(\psi)}{\Gamma(\psi \to \mu^+\mu^-)} = a(m_c)[1 + (6.66 - 0.18f)a(m_c)\,],$$

where f is the flavour number. Experimental values are equal[10] to $(4\alpha/5\pi) \times 38.7$ and 0.21, respectively, with uncertainties which we put equal to $\pm 10\%$ on methodical grounds.

We use the following theoretical approaches:

(a) The second (next-to-leading) order RG perturbation theory $R_{RG} = a(1 + ar_1)$, where a is defined as a solution of the Eq.

$$\beta_1 \ln \frac{M^2}{\Lambda^2} = \frac{1}{a} - \rho \ln\left(1 + \frac{1}{a\rho}\right); \quad \rho = \frac{\beta_2}{\beta_1}. \tag{8}$$

(b) The scheme-invariant perturbation theory in 2-loop order combining Eqs.(5) and (6) yields R as a solution of

$$\beta_1 \ln \frac{M^2}{\Lambda^2} - r_1 = \frac{1}{R} - \rho \ln\left(1 + \frac{1}{\rho R}\right). \tag{9}$$

With the help of Eqs.(8) and (9) we obtain numerical values of $\Lambda_{\overline{MS}}$ presented in Fig.1, where error bars correspond to 10% data uncertainties.

We have also used for these two examples:

(c) The Stevenson optimization procedure[11]. To a given order of PT it corresponds to the introduction of one additional parameter ξ in the RG solution

$$R_{RG}\left(\frac{Q^2}{\mu^2}, \alpha\right) = R\left(\xi, \alpha\left(\frac{Q^2}{\xi\mu^2}, \alpha\right)\right)$$

in such a manner that $r_1(\xi) = r_1(1) + \beta_1 \ln \xi$.

Fig.1

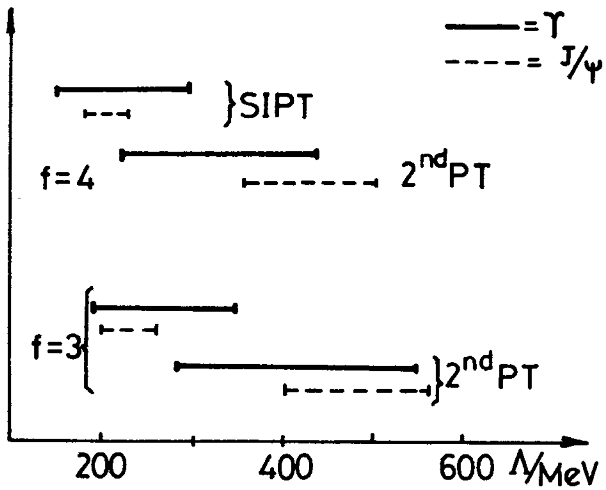
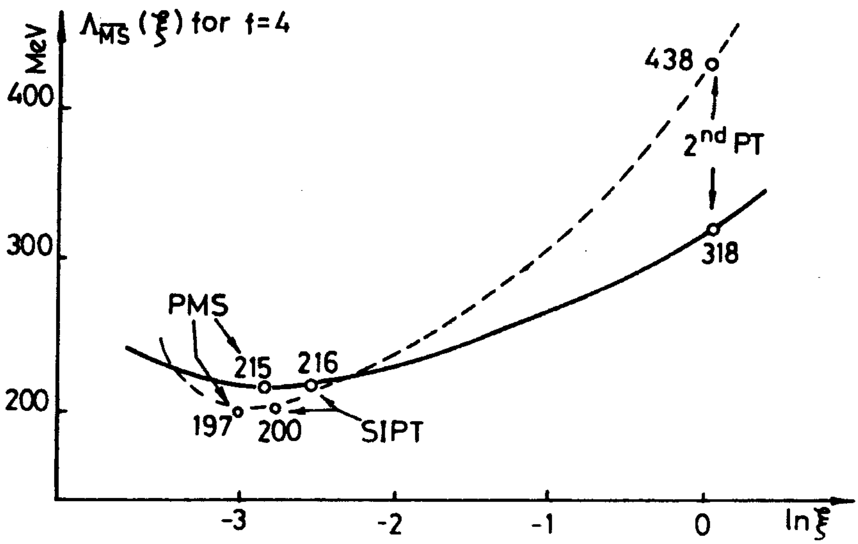


Fig.2



The Stevenson"Procedure of Minimal Sensitivity" (PMS) determines the $\xi = \xi_0$ values from the condition $\Lambda'(\xi_0) = 0$. The "physical" value is then equal to $\Lambda(\xi_0)$.

In Fig.2 the combination of results obtained by (a), (b) and (c) procedures are given (for f = 4). Curves represent $\Lambda(\xi)$ dependences. Marks "2nd PT" correspond to (a) procedure, "SI" - to (b) and "PMS" to (c).

It follows from this picture that SIPT gives results very close to the Stevenson PMS recipe but in contrast to it provides a simple analytical algorithm.

Note also that according to Fig.1 final results depend on the flavour number f. So, e.g., the mean values of $\Lambda^{J/\psi}_{\overline{MS}}$ = 230 MeV and 200 MeV for f = 3 and f = 4 respectively. Thus, it is evident that at the moment the theoretical "flavour number uncertainty" is of the same order as the experimental one.

Our next step is to consider this problem in the framework of SIPT.

3. The Account of Mass Dependence

First of all we have to fix the way of mass renormalization. We choose the pole mass most suitable for our programme. In other words we fix the subtraction procedure so that the fermion self-energy operator is subject to the condition $\Sigma(\hat{p} = m) = 0$. It turns out that this is the only subtraction we really need in the course of calculations.

The result of the usual quantum field perturbation theory for the R-function now has the structure

$$R_{PT}(Q^2, \ldots) = a_i - a_i^2 U_1^i (Q^2, m^2) + $$
$$+ a_i^3 [(U_1^i)^2 - U_2^i (Q^2, m^2)] + O(a_i^4).$$

Note that out notation (according to [12]) includes the case of "no-charge renormalization", where $a = a_{BARE}$ and U_ℓ^{BARE} contains the singular dependence of regularization parameter.

Differentiating with respect to $\ln Q^2$ and reexpressing a_i as a function of R and U_ℓ^i we obtain quite similarly to the massless case eq.(4) with SI coefficients f_ℓ depending on Q^2/m^2. After integrating this Eq. in 1-loop approximation ($\ell_{max} = 1$) we obtain

$$R^{(1)} (Q^2, \ldots) = \frac{1}{u_1(Q^2, \ldots)}, \tag{10}$$

where u_1 differs from any of U_1^i by a constant $u_1 = U_1^i + c_1^i$ so that u_1 is an SI quantity depending on one free parameter, which can be chosen as a scale Λ_R or "R-process coupling constant". In the typical case

$$U_\ell^i (Q^2, \ldots) = \beta_\ell(3) \ln\frac{Q^2}{\mu^2} - \sum_h I_\ell \left(\frac{Q^2}{m_n^2}\right) + V_\ell^i, \tag{11}$$

where $I_\ell(x)$ are heavy-quark contributions with properties $I_\ell(0) = 0$, $I_\ell(x) \to c_\ell \ln x + d_\ell$, as $x \to \infty$, and V_ℓ^i is a constant (possibly singular) depending on the renormalization scheme.

At the same time u_1 instead of SD constant V_1^i contains SI constant and can be represented in the form

$$u_1(Q^2, m^2) = \beta_1(3)\ln\frac{Q^2}{\Lambda^2_{R3}} - \sum_h I_1(\frac{Q^2}{m_h^2}). \qquad (12)$$

Here Λ_{R3} plays the role of "a scale parameter of R-process in the 3-flavour region". It is suitable for the discussion of confinement physics. On the other hand, one can choose as well

$$u_1(Q^2, m^2) = \beta_1(6)\ln\frac{Q^2}{\Lambda^2_{R6}} - \sum_h \tilde{I}_1(\frac{Q^2}{m_h^2}).$$

where $\tilde{I}_1(x) = I_1(x) - \frac{2}{3}\ln x - d_1 \to 0$ as $x \to \infty$. The latter choice is more convenient for extrapolation of it into a very high energy region, say, into the domain of GUT. The simple and convenient choice of integration constant is provided by the relation $u_1(Q^2, m^2) = 0$ for $Q^2 = \Lambda^2$:

$$u_1(Q^2, m^2) = \beta_1(3)\ln\frac{Q^2}{\Lambda^2} - \sum_h \{ I_1(\frac{Q^2}{m_h^2}) - I_1(\frac{\Lambda^2}{m_h^2})\}. \qquad (13)$$

The mass-dependent result (10) is a generalization of the massless one-loop formula. In the 2-loop case the corresponding expression can also be obtained. We get[6] instead of (10):

$$R^{(2)}(Q^2, m^2) = [u_1(Q^2, m^2) + \frac{u_2(Q^2, m^2)}{u_1(Q^2, m^2)}\ln u_1(Q^2, m^2)]^{-1}, \qquad (14)$$

where u_2 corresponds to a pure 2-loop contribution U_2^i in the same way as u_1 to U_1. The relation between parameters for different processes has the form analogous to the massless case:

$$\beta_1(3)\ln(\frac{\Lambda_A}{\Lambda})^2 = -\sum_h \{I_{1B}(\frac{\Lambda_B^2}{m_h^2}) - I_{1A}(\frac{\Lambda_A^2}{m_h^2})\} + V_{1B}^i - V_{1A}^i. \qquad (15)$$

while the corresponding relation between SI parameter Λ_R and SD parameter Λ^i for a given process R looks like:

$$\beta_1(3)\ln(\frac{\Lambda^i}{\Lambda_R})^2 = -\sum_h I_1(\frac{\Lambda_R^2}{m_h^2}) + V_1^i. \qquad (16)$$

Eqs. (15), (16) correspond to the choice of u_1 as in eq. (13).

For the functions with anomalous dimensions

$$\Gamma_{PT}(Q^2, m^2, ...) = 1 + a_i S_1(Q^2, m^2) + a_i^2 S_2^i(Q^2, m^2) + ...$$

in the 1-loop case we obtain the resulting expression

$$\Gamma^{(1)}(Q^2, m^2) = \Gamma_0 [u_1^\Gamma(Q^2, m^2, \Lambda_\Gamma^2)]^{n(Q^2)}, \quad n = \frac{s_1}{u_1^\Gamma}, \tag{17}$$

where s_1 is the SI function corresponding to the 1-loop contribution S_1^i and

$$(u_1^\Gamma)^{-1} = R_\Gamma \equiv a_i + a_i^2 \left\{ \frac{(S_2^i)'}{(S_1^i)'} - S_1^i \right\}$$

is the dimensionless amplitude of R-type (an analog of the effective coupling) specific to the considered Γ-amplitude. (All this construction is analogous to the massless case described in detail in paper /12/).

Here the only SD quantity is the constant factor Γ_0 It is possible to write down the corresponding 2-loop expression parallel to the 2-loop generalization of Eq.(1). (See, e.g., Eq.(20) in ref. /6/).

4. Conclusion

Our results consist of the set of Eqs.(14)-(17).They represent a massive generalization of the SIPT-formalism and contain threshold dependence via explicit functions I_ℓ, S_ℓ which can be calculated in perturbation theory. These final expressions,quite analogous to massive SD results of paper /6/, do not contain famous renormgroup β and γ functions, but just perturbative coefficients U_ℓ and S_ℓ.

The equations obtained can be used in QCD for the SI description of the Q^2 behaviour in the regions close to heavy quark creation threshold. They provide a continuous analytic interpolation between massless (logarithmic) expressions with different values of the flavour number.

It is interesting to note that the attractive idea of getting rid of SD from the renormalized quantum field theory can be considered without appeal to the renormgroup. This idea applied to usual perturbation expansions can be realized for elements of S-matrix and higher vertices which do not allow the renormgroup treatment.

It is rather evident that every such function Γ can be expressed in the SI way in terms of its own value Γ_0 taken at some fixed values of kinematical arguments. This can be done even out of the perturbation framework. However, the equation relating "coupling constants of different processes" seems to be simple only in perturbation theory.

References

1. Bogolubov N.N., Shirkov D.V. Doklady AN SSSR, 1955, vol.103, p.203.
2. Bogolubov N,N., Shirkov D.V. Nuovo Cim., 1956, vol.3, p.845.
3. Krasnikov N.V. CERN Preprint, TH-2119, Geneva, 1980.
4. Grunberg G. Phys.Lett., 1980, vol.95B, p.70.
5. Dhar A. Phys.Lett., 1983, vol.128B, p.407.
6. Shirkov D.V. Teor.Mat.Fiz., 1981, vol.49, p.291; also JINR, E2-81-274, Dubna, 1981 (in English).
7. Blank V.Z., Shirkov D.V.Nucl.Phys., 1956, vol.2, p.356.
8. Bogolubov N.N., Shirkov D.V. Introduction to the Theory of Quantized Fields. Inters.Pub., N.Y., 1959, Ch.8.
9. Mackenzie P.B., Lepage G.P.Phys.Rev.Lett., 1981, vol.47, p.1244; Buchmuller W., Ng Y.J., Tye S.-H.H. Phys. Rev., 1981, vol.D24, p.3003.
10. Berkelman K. In: Proc. XX Int.Conf.on High Energy Phys., Madison, 1980; Durand L., Pondrom L.G., eds. AIP, N.Y., 1981, p.1499.
11. Stevenson P.M. Phys.Rev., 1981, vol.D23, p.2916.
12. Dhar A., Gupta V. A New Perturbative Approach to Renormalizable Theories. Tata Inst. preprint TIFR/TH/83-18, 1983.

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